Hands-on Gravitational Wave Astronomy: Extracting astrophysical information from simulated signals

Louis J. Rubbo

Center for Gravitational Wave Physics, Pennsylvania State University, University Park, PA 16802

Shane L. Larson*

Department of Physics, Weber State University, Ogden, UT 84408

Michelle B. Larson*

Department of Physics, Utah State University, Logan, UT 84322

Dale R. Ingram

LIGO Hanford Observatory, Richland, WA 99352

(Dated: February 2, 2008)

In this paper we introduce a hands-on activity in which introductory astronomy students act as gravitational wave astronomers by extracting information from simulated gravitational wave signals. The process mimics the way true gravitational wave analysis will be handled by using plots of a pure gravitational wave signal. The students directly measure the properties of the simulated signal, and use these measurements to evaluate standard formulae for astrophysical source parameters. An exercise based on the discussion in this paper has been written and made publicly available online for use in introductory laboratory courses.

I. INTRODUCTION

Observational astronomy stands at the threshold of an era where gravitational wave detectors are a tool which regularly contributes important information to the growing body of astrophysical knowledge.[1] Ground based detectors such as the Laser Interferometer Gravitational-wave Observatory[2] (LIGO) and the forthcoming space based detector the Laser Interferometer Space Antenna[3] (LISA) will probe different regimes of the gravitational wave spectrum and observe sources that radiate at different gravitational wavelengths. Unlike their cousins, traditional electromagnetic telescopes, gravitational wave detectors are not imaging instruments. How then does a gravitational wave astronomer take the output from a detector and extract astrophysical information about the emitting sources? This paper introduces a hands-on activity in which introductory astronomy students answer this question.

Traditional astronomy is often presented through the medium of colorful images taken with large scale telescopes. In addition to studying images, astronomers learn about astrophysical systems by collecting data at multiple wavelengths, using narrow band spectra, measuring time varying light curves, and so on. It is often the case that the core physics governing the evolution of these distant systems is deduced from the physical character of the observed electromagnetic radiation, rather than from the imagery that is used to illustrate the science for other audiences.

Gravitational wave astronomy is completely analogous to its electromagnetic cousin, with one important distinction: there will be no image data, because gravitational wave detectors are not imaging instruments. Gravitational wave observatories like LIGO and LISA return a noisy time series that has encoded within it gravitational wave signals from one or possibly many overlapping sources. To gain information about the systems emitting these gravitational wave signals requires the use of time series analysis techniques such as Fourier transforms, Fisher information matrices, and matched filtering. Recently an activity has shown how students can emulate the match filtering process by comparing ideal signals to mocked noisy detector output.[4] In this paper, a procedure is described whereby students can analyze a simulated gravitational wave signal and extract the astrophysical parameters which describe the radiating system. The goal is to introduce students to how gravitational wave astronomers learn about sources of gravitational radiation in a fashion suitable for classroom or laboratory exercises related to this modern and emerging branch of observational astrophysics.

The rest of this article introduces some basic background of gravitational radiation and then describes the activity. Section II outlines the theory connecting the structure of gravitational waves to astrophysical parameters, and Section III illustrates the characteristic waveforms from a typical binary system. Section IV illustrates a procedure where measurements made from waveform plots, together with the theory of waveform generation, can be used to extract the astrophysical parameters (orbital period, distance, etc.) of the system emitting gravitational radiation. Section V discusses implementations and extensions for this activity in an introductory astronomy course. The analysis described in the paper has been implemented in an activity format, complete with a keyed solution for the instructor, and is publicly available online.[5]

II. GRAVITATIONAL WAVE PRODUCTION IN BINARIES

In electromagnetism radiation is produced by an accelerating charged particle. Similarly in general relativity, gravitational radiation is produced by an accelerating mass. To be precise, gravitational waves are produced by a time varying mass quadrupole moment. The reason for this is straightforward.[6] Monopole radiation is prevented due to conservation of mass, while dipole radiation does not occur due to the conservation of momentum. This leaves the quadrupole as the leading order term in the multipole expansion of the radiation field. A simple and common example of an astrophysical system with a time varying quadrupole moment is a binary star system.

For a circular binary system, where the components are treated as point-like particles, the gravitational waveforms take on the seductively simple form

$$h(t) = \mathcal{A}(t)\cos(\Phi(t))\,, \tag{1}$$

where $h(t)$ is the gravitational waveform (also referred to as the gravitational wave strain), $\mathcal{A}(t)$ is the time dependent amplitude, and $\Phi(t)$ is the gravitational wave phase. The amplitude $\mathcal{A}(t)$ can be expressed in terms of the physical parameters characterizing the system,

$$\mathcal{A}(t) = \frac{2(G\mathcal{M})^{5/3}}{c^4 r}\left(\frac{\pi}{P_{gw}(t)}\right)^{2/3}\,, \tag{2}$$

where G is Newton's gravitational constant, c is the speed of light, r is the luminosity distance to the binary, and $P_{gw}(t)$ is the gravitational wave period. The quantity $\mathcal{M} \equiv (M_1 M_2)^{3/5}(M_1+M_2)^{-1/5}$ is called the chirp mass and appears repeatedly in gravitational wave physics, making it a natural mass scale to work with. The origin for this nomenclature will become evident shortly. The waveform phase $\Phi(t)$ is given by a the integral:

$$\Phi(t) = \Phi_0 + 2\pi\int_0^t \frac{dt'}{P_{gw}(t')}\,, \tag{3}$$

where Φ_0 is the initial phase.

As in electromagnetism, gravitational waves have two independent polarization states. For a binary system the two states are related by a 90° phase shift. Consequently, Eq. (1) captures the functional form for both polarization states. For the purposes of this paper only a single polarization state and its associated waveform will be discussed.

Gravitational waves carry energy and angular momentum away from the binary system causing the orbital period to decrease with time according to[7]

$$P_{orb}(t) = \left(P_0^{8/3} - \frac{8}{3}kt\right)^{3/8}\,, \tag{4}$$

where P_0 is the orbital period at time $t = 0$, and k is an evolution constant given by

$$k \equiv \frac{96}{5}(2\pi)^{8/3}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3}\,. \tag{5}$$

As a consequence of the ever shortening orbital period, the two binary components will slowly inspiral, eventually colliding and coalescing into a single remnant. Under the assumption of point-like particles made here, this formally occurs when $P_{orb}(t) = 0$.

Note that Eq. (4) gives the orbital period *not* the gravitational wave period. Careful scrutiny of Eqs. (2) and (3) will reveal that the gravitational wave period $P_{gw}(t)$ is the quantity which appears in the description of the waveform. Fortunately, for circularized binary systems, $P_{orb}(t)$ and $P_{gw}(t)$ are simply related:

$$P_{orb}(t) = 2P_{gw}(t)\,. \tag{6}$$

The simple factor of two stems from the fact that the lowest possible order for gravitational radiation production is the quadrupole order. Moreover, quadrupole moments are invariant under a 180° rotation, yielding a factor of two per complete orbit.

III. WAVEFORMS FROM A BINARY SYSTEM

As an illustrative example of the kind of waveforms we expect from binaries, consider a binary neutron star system with $M_1 = M_2 = 1.4\ \mathrm{M}_\odot$ ($\mathcal{M} = 1.22 M_\odot$) located at the center of the galaxy $r = 8$ kpc away. For the activity we will consider waveforms generated at two distinct times in the binary's evolution. The first waveform we will consider is $\sim 10^6$ years before coalescence. During this phase the gravitational wave frequency is in the regime that will be detectable by the spaceborne LISA observatory, which has a principle sensitivity in the range of 10^{-5} Hz to 1 Hz. The second waveform considered will be during the final second before the neutron star binary coalesces. The gravitational wave frequencies during this phase are in the regime that will be detectable by the terrestrial LIGO observatory, which is sensitive to gravitational wave frequencies between 10 Hz and 10^3 Hz.

A. Far from Coalescence

Figure 1 shows the emitted gravitational radiation long before the binary components coalesce. During this era of the binary evolution, the gravitational waves are essentially monochromatic; the orbital period is evolving too slowly to detect a frequency derivative term.

For monochromatic signals like this, the only measurable properties of the gravitational waveform are the period P_{gw} (and the orbital period P_{orb} through Eq. (6)), amplitude $\mathcal{A}$, and initial phase Φ_0. Even though the waveform equations depend on the chirp mass $\mathcal{M}$ and the luminosity distance r, it is not possible to solve for their values from the data provided by the monochromatic waveform. Not enough information exists to completely solve Eqs. (2) and (4) together for both quantities. This can be seen by considering the relative size

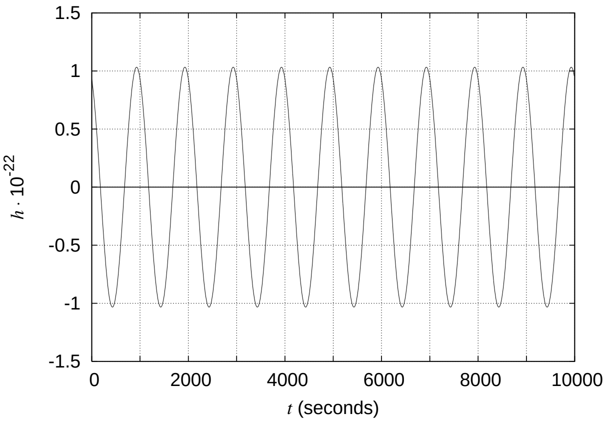

FIG. 1: The gravitational waveform for a binary system consisting of two neutron stars far from coalescence and located at the center of our galaxy.

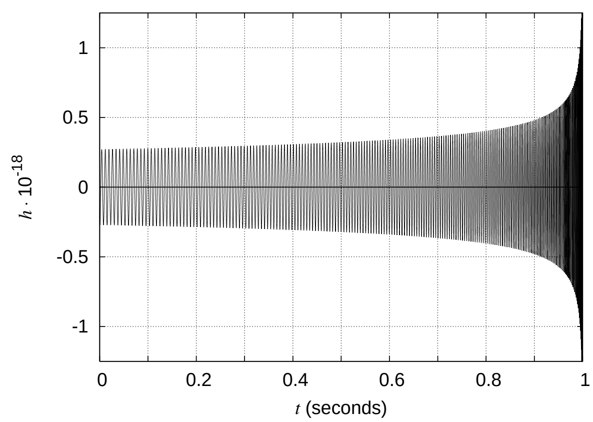

FIG. 2: The waveform over the last second before coalescence. Since the signal's amplitude and frequency is increasing with time, these types of systems are said to be *chirping*.

of the two terms in Eq. (4); using the binary neutron star chirp mass $\mathcal{M} = 1.22\ \mathrm{M}_\odot$, it should be evident that the second term is completely negligible compared to the period P_0 of the wave shown in Fig. 1. In the parlance of gravitational wave astronomy, there is a *mass-distance degeneracy* in the waveform description, analogous to the familiar mass-inclination degeneracy in the electromagnetic observations of spectroscopic binaries. This degeneracy is a well known problem, but as the next section shows, it can be broken if the orbital period of the binary evolves during the gravitational wave observations.

B. Near Coalescence

Inspection of Eq. (4) shows that as time goes on, the emission of gravitational waves causes the orbital period to grow shorter, and as a result the frequency of the emitted waves increases. Similarly, consideration of Eq. (2) shows that as the wave period decreases, the time dependent amplitude $\mathcal{A}(t)$ increases. This is characteristic behavior for gravitational waves emitted just prior to a source coalescence, and is known as a *chirp*. The chirp waveform emitted by the example binary neutron star system just prior to coalescence is illustrated in Fig. 2.

Any binary signal which evolves appreciably during the gravitational wave observation is called a chirping binary. In these cases, the mass parameter $\mathcal{M}$ which appears in the amplitude $\mathcal{A}(t)$ and in the period evolution constant k can be determined from measurements of the evolving signal. For this reason, the mass $\mathcal{M}$ is called the *chirp mass*. To leading order in gravitational wave production, it is not possible to measure the individual masses, only the chirp mass. Consequently, it is not possible to distinguish between binaries with the same chirp mass. For example, the binary neutron star considered in this paper with $M_1 = M_2 = 1.4\ \mathrm{M}_\odot$ has roughly the same chirp mass as a binary with an $M_1 = 10\ \mathrm{M}_\odot$ black hole and a $M_2 = 0.3\ \mathrm{M}_\odot$ white dwarf.

To extract the chirp mass from measurements of the gravitational waveform, consider two small stretches of the chirping waveform. Figure 3 shows the waveform from $0\ \mathrm{s} \leq t \leq 0.05$ s, and Fig. 4 shows the waveform from $0.9\ \mathrm{s} \leq t \leq 0.92$ s. The waveform is appreciably different between these two snapshots, both in amplitude $\mathcal{A}(t)$ and in period $P_{gw}(t)$. This allows the degeneracy found in the monochromatic signal case to be broken, because the gravitational wave period can be measured at two different times and used in Eq. (4) to solve for the chirp mass $\mathcal{M}$.

IV. MEASURING GRAVITATIONAL WAVEFORMS

This section illustrates a procedure at the introductory astronomy level where students can make direct measurements from the figures in Section III using a straight edge and the axis labels. Using their measured data together with the theory presented in Section II, the astrophysical character of the system emitting the gravitational waveforms can be deduced.

A. Monochromatic Waveforms

Limited astrophysical information can be extracted directly from Fig. 1, as will be the case with true monochromatic signals detected by gravitational wave observatories. With limited assumptions more detailed information can be deduced, which will be valid so long as the assumptions are valid. A suitable extraction and analysis procedure for an introductory astronomy student would proceed in the following manner:

- The gravitational wave period P_{gw} can be measured directly from the figure. Since the signal is monochromatic, the binary is circular and the orbital period P_{orb} is obtained directly from P_{gw} using Eq. (6). For the waveform in Fig. 1 careful measurement should yield a value of $P_{gw} = 1000$ sec.

- The amplitude $\mathcal{A}$ and the initial phase Φ_0 can also be measured directly from the figure. As noted in Section III A no astrophysical information can be extracted from the amplitude alone. The initial phase is a simple quantity to measure, but does not represent any intrinsic property of the binary; its value is solely a consequence of when the gravitational wave observations began. To illustrate this, imagine relabeling the time axis in Fig. 1 to represent a new observation which started somewhat later than the observation shown. The initial phase will have some new value, but the waveform itself does not change because the intrinsic properties of the binary did not change.

- If a gravitational wave astronomer were to assume that the binary was a pair of neutron stars, the component mass values could be assigned as part of the assumption. Most neutron star masses cluster around $M = 1.4\ \mathrm{M}_\odot$, so a good base assumption is that each component of the binary has this mass. As noted in Section III A this assumption can be a dangerous one, since similar chirp masses $\mathcal{M}$ can result from significantly different systems. Other information, not present in the gravitational waveform, may help an astronomer feel more confident about such an assumption. For example, an associated simultaneous electromagnetic signal or the location of the source on the sky may favor one model of the binary over another.

- If the masses are assumed, the orbital separation of the binary components, R, can be computed from the measured orbital period by using Kepler's Third Law:

$$G(m_1 + m_2) = \left(\frac{2\pi}{P_{orb}}\right)^2 R^3 \,. \tag{7}$$

  For this example, the orbital separation is $R = 1.4 \times 10^{-3}$ AU $= 2.1 \times 10^8$ m, or a little less than the separation of the Earth and the Moon.

- If the masses are assumed, the distance to the binary can be computed from Eq. (2) and the measured amplitude. If careful measurements have been made, the answer should be close to the value $r = 8$ kpc $= 2.5 \times 10^{20}$ m.

- Lastly, if the masses are assumed, it can be quantitatively shown that the monochromatic descriptor is a good one for this wave by computing the value of the second term in Eq. (4) and showing that it is negligible compared to the measured period P_0.

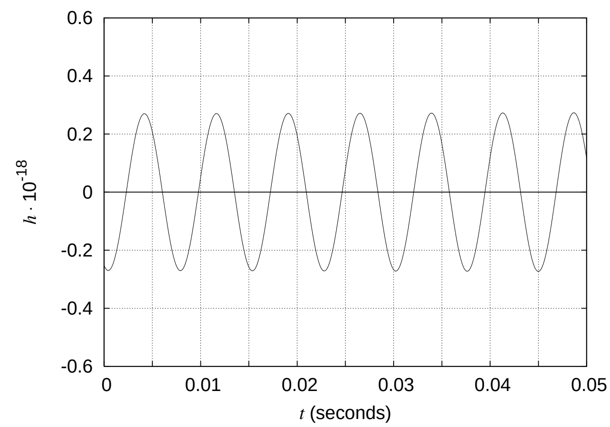

FIG. 3: The chirping waveform one second before coalescence.

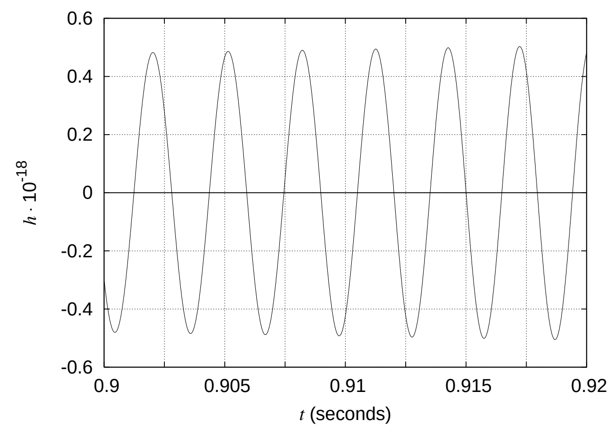

FIG. 4: The chirping waveform one-tenth of a second before coalescence.

B. Chirping Waveforms

In the case of a chirping waveform, additional astrophysical information associated with the system can be extracted directly from measurements of the waveform without making underlying assumptions like those needed when the system was far from coalescence. To extract information from the chirping waveform shown in Fig. 2, the two zoom-ins of the waveform shown in Figs. 3 and 4 will be used. A typical extraction procedure might look like this:

- For each of the Figs. 3 and 4, measure the period of one cycle of the wave, and note the time t at which the periods were measured. The amplitudes $\mathcal{A}(t)$ should be measured for the same cycle as the periods.

- If the period measured at time t_1 in Fig. 3 is P_0, and the period measured at time t_2 in Fig. 4 is $P_{gw}(t)$ at time $t = t_2 - t_1$, then Eq. (4) can be used to deduce the chirp mass $\mathcal{M}$ of the system.

- Once the chirp mass $\mathcal{M}$ has been determined, the distance to the binary can be computed by using Eq. (2) with the measured amplitude $\mathcal{A}(t)$ and period $P_{gw}(t)$ of each waveform. The results from the two figures can be averaged together to obtain a final result.

V. DISCUSSION

This paper introduced the core calculations in gravitational wave astrophysics an introductory astronomy student can perform in a laboratory setting to glean information about an astrophysical system. To compliment this article we have also developed a student activity sheet and corresponding teacher's guide related to the exercises described in sections IV A and IV B. The complimentary material is available, along with a template activity,[4] at `http://cgwp.gravity.psu.edu/outreach/activities/`.

The activity described here is a simple introduction to how gravitational wave astronomers extract astrophysical information from observed binary waveforms. Real signal analysis is a more complex endeavor than what has been presented here. The most significant challenge in the case of true data is identifying the signal buried in a noisy data stream. A common approach to this problem in gravitational wave astronomy is to use template matching, which has been explored in a separate activity.[4] If a signal is present in a noisy data stream, the template provides a way to subtract the noise away and leave a clean waveform behind. This is the assumed starting point in the activity developed here. Its from clean waveforms that astronomers will estimate the values of astrophysical parameters describing a source of gravitational waves. By sequencing the two activities a student is exposed, at least in an idealized way, to the methods used by gravitational wave astronomers to extract astrophysical information about the emitting systems.

Acknowledgments

This work was supported by the Center for Gravitational Wave Physics. The Center for Gravitational Wave Physics is funded by the National Science Foundation under cooperative agreement PHY-01-14375. The authors would also like to thank the LIGO Laboratory. LIGO is funded by the National Science Foundation under Cooperative Agreement PHY-0107417.

---

[*] Formerly at the Center for Gravitational Wave Physics, Pennsylvania State University, University Park, PA 16802

[1] L. J. Rubbo, S. L. Larson, M. B. Larson, and K. D. Kaleski, "Gravitational Waves: New Observatories for New Astronomy," Phys. Teach. **44**, 420-423 (2006).

[2] A. Abramovici *et al.*, "LIGO: The Laser Interferometer Gravitational-Wave Observatory," Science **256**, 325-333 (1992).

[3] T. J. Sumner and D. N. A. Shaul, "The Observations of Gravitational Waves from Space using LISA," Mod. Phys. Lett. A **19**, 785-800 (2004).

[4] M. B. Larson, L. J. Rubbo, K. D. Zaleski, and S. L. Larson, "Science Icebreaker Activities: An Example from Gravitational Wave Astronomy," Phys. Teach. **44**, 416-419 (2006).

[5] `http://cgwp.gravity.psu.edu/outreach/activities/`.

[6] B. F. Schutz, "Gravitational Waves on the Back of an Envelope," Am. J. Phys. **5**, 412-419 (1984).

[7] J. B. Hartle, *Gravity: An Introduction to Einstein's General Relativity* (Addison Wesley, San Francisco, 2003), pp. 508